Mark Scheme (Results)

Autumn 2020

Pearson Edexcel GCE

In Mathematics (9MA0) Paper 2

Pure Mathematics 2

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Publications Code 9MA0_02_2010_MS

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 100.

2. The Edexcel Mathematics mark schemes use the following types of marks:
   - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
   - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)
   - Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol √ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- ⎡ The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses <u>and indicates which response they wish to submit</u>, examiners should mark this response.
   If there are several attempts at a question <u>which have not been crossed out</u>, examiners should mark the final answer which is the answer that is the <u>most complete</u>.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

General Principles for Further Pure Mathematics Marking

(*But note that specific mark schemes may sometimes override these general principles*)

<u>Method mark for solving 3 term quadratic:</u>

1. Factorisation

$(x^2+bx+c)=(x+p)(x+q)$, where $|pq|=|c|$, leading to $x=\ldots$

$(ax^2+bx+c)=(mx+p)(nx+q)$, where $|pq|=|c|$ and $|mn|=|a|$, leading to $x=\ldots$

2. Formula

Attempt to use the correct formula (with values for *a*, *b* and *c*)

3. Completing the square

Solving $x^2+bx+c=0$: $\left(x\pm\frac{b}{2}\right)^2\pm q\pm c=0,\quad q\neq 0$, leading to $x=\ldots$

<u>Method marks for differentiation and integration:</u>

1. Differentiation

Power of at least one term decreased by 1. $(x^n\to x^{n-1})$

2. Integration

Power of at least one term increased by 1. $(x^n\to x^{n+1})$

<u>Use of a formula</u>

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

<u>Method mark</u> for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is <u>not</u> quoted, the method mark can be gained by implication from <u>correct</u> working with values but may be lost if there is any mistake in the working.

<u>Exact answers</u>

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question	Scheme	Marks	AOs
1(a)	$h = 0.5$	B1	1.1a
	$A \approx \frac{0.5}{2}\left\{0.5774 + 0.8452 + 2\left(0.7071 + 0.7746 + 0.8165\right)\right\}$	M1	1.1b
	= awrt 1.50	A1	1.1b
	For reference: **The integration on a calculator gives 1.511549071** **The full accuracy for *y* values gives 1.504726147** **The accuracy from the table gives 1.50475**		
		(3)	

(b)	$3 \times$ their (a)
	If (a) is correct, allow awrt 4.50 or awrt 4.51 even with no working.
	Only allow 4.5 if (a) is correct and working is shown e.g. 3×1.5
	
	If (a) is incorrect allow $3 \times$ their (a) given to at least 3sf but do not be too concerned about the accuracy (as they may use rounded or rounded value from (a))
		B1ft	2.2a

	For reference the integration on a calculator gives 4.534647213		
		(1)	
(c)	**<u>This mark depends on the B1 having been awarded in part (b) with awrt 4.5</u>** Look for a sensible comment. Some examples: • The answer is accurate to 2 sf or one decimal place • Answer to (b) is accurate as $4.535 \approx 4.50$ • Very accurate as 4.535 to 2 sf is 4.5 • $4.51425 < 4.535$ so my answer is underestimate but not too far off • It is an underestimate but quite close • It is a very good estimate • High accuracy • (Quite) accurate • It is less than 1% out • $4.535 - 4.5 = 0.035$ so not far out But **not** just "it is an underestimate" or Calculates percentage error correctly using awrt 4.50 or awrt 4.51 or 4.5 (No comment is necessary in these cases although one may be given) Examples: $\left\lvert\frac{4.535-4.50}{4.535}\right\rvert\times100 = 0.77\%$ or $\left\lvert\frac{4.535-4.51}{4.535}\right\rvert\times100 = 0.55\%$ or $\left\lvert\frac{4.535-4.51425}{4.535}\right\rvert\times100 = 0.46\%$ or $\left\lvert\frac{4.50}{4.535}\right\rvert\times100 = 99\%$ In these cases don't be too concerned about accuracy e.g. allow 1sf. **This mark should be withheld if there are any contradictory statements**	B1	3.2b
		(1)	
			(5 marks)

Notes:

(a)

B1: States or uses h = 0.5. May be implied by $\frac{1}{4}\times\{...$ below.

M1: Correct attempt at the trapezium rule.

Look for $\frac{1}{2}h\times\{0.5774+0.8452+2(0.7071+0.7746+0.8165)\}$ condoning slips on the terms but must use all y values with no repeats.

There must be a clear attempt at $\frac{1}{2}h\times$(first y + last y + $2\times$"sum of the rest")

Give M0 for $\frac{1}{2}\times\frac{1}{2}0.5774+0.8452+2(0.7071+0.7746+0.8165)$ unless the missing brackets are implied.

NB this incorrect method gives 5.85…

May be awarded for separate trapezia e.g.

$$\frac{1}{4}(0.5774+0.7071)+\frac{1}{4}(0.7071+0.7746)+\frac{1}{4}(0.7746+0.8165)+\frac{1}{4}(0.8165+0.8452)$$

May be awarded for using the function e.g. $\frac{1}{2}h\times\left\{\sqrt{\frac{0.5}{1+0.5}}+\sqrt{\frac{2.5}{1+2.5}}+2\left(\sqrt{\frac{1}{1+1}}+\sqrt{\frac{1.5}{1+1.5}}+\sqrt{\frac{2}{1+2}}\right)\right\}$

A1: Awrt 1.50 (Apply isw if necessary)

Correct answers with no working – send to review

(b)

B1ft: See main scheme. Must be considering 3 × (a) and not e.g. attempting trapezium rule again.

(c)

B1: See scheme

Question Number	Scheme	Marks	AO's
2	Attempts any one of $\left(\pm\overrightarrow{PQ}=\right)\pm(\mathbf{q}-\mathbf{p}),\ \left(\pm\overrightarrow{PR}=\right)\pm(\mathbf{r}-\mathbf{p}),\ \left(\pm\overrightarrow{QR}=\right)\pm(\mathbf{r}-\mathbf{q})$ Or e.g. $\left(\pm\overrightarrow{PQ}=\right)\pm\left(\overrightarrow{OQ}-\overrightarrow{OP}\right),\ \left(\pm\overrightarrow{PR}=\right)\pm\left(\overrightarrow{OR}-\overrightarrow{OP}\right),\ \left(\pm\overrightarrow{QR}=\right)\pm\left(\overrightarrow{OR}-\overrightarrow{OQ}\right)$	M1	1.1b
	Attempts e.g. $\mathbf{r}-\mathbf{q}=2(\mathbf{q}-\mathbf{p})$ $\mathbf{r}-\mathbf{p}=3(\mathbf{q}-\mathbf{p})$ $\frac{2}{3}(\mathbf{q}-\mathbf{p})=\frac{1}{3}(\mathbf{r}-\mathbf{q})$ $\mathbf{q}=\mathbf{p}+\frac{1}{3}(\mathbf{r}-\mathbf{p})$ $\mathbf{q}=\mathbf{r}+\frac{2}{3}(\mathbf{p}-\mathbf{r})$	dM1	3.1a
	E.g. $\Rightarrow \mathbf{r}-\mathbf{q}=2\mathbf{q}-2\mathbf{p}\Rightarrow 2\mathbf{p}+\mathbf{r}=3\mathbf{q}\Rightarrow \mathbf{q}=\frac{1}{3}(\mathbf{r}+2\mathbf{p})$ *	A1*	2.1
		(3)	
			(3 marks)

Notes:

M1: Attempts any of the relevant vectors by subtracting either way around. This may be implied by sight of any one of $\pm(\mathbf{q}-\mathbf{p}),\ \pm(\mathbf{r}-\mathbf{p}),\ \pm(\mathbf{r}-\mathbf{q})$ ignoring how they are labelled

dM1: Uses the given information and writes it correctly in vector form that if rearranged would give the printed answer

A1*: Fully correct work leading to the given answer. Allow OQ = … as long as OQ has been defined as **q** earlier.

In the working allow use of P instead of **p** and Q instead of **q** as long as the intention is clear.

Question	Scheme	Marks	AOs

3(a)	$2\log(4-x)=\log(4-x)^2$	B1	1.2
	$2\log(4-x)=\log(x+8)\Rightarrow\log(4-x)^2=\log(x+8)$ $(4-x)^2=(x+8)$ **or** $2\log(4-x)=\log(x+8)\Rightarrow\log(4-x)^2-\log(x+8)=0$ $\dfrac{(4-x)^2}{(x+8)}=1$	M1	1.1b
	$16-8x+x^2=x+8\Rightarrow x^2-9x+8=0$ *	A1*	2.1
		(3)	
	(a) Alternative - working backwards:		
	$x^2-9x+8=0\Rightarrow(4-x)^2-x-8=0$	B1	1.2
	$\Rightarrow(4-x)^2=x+8$ $\Rightarrow\log(4-x)^2=\log(x+8)$	M1	1.1b
	$\Rightarrow 2\log(4-x)=\log(x+8)$ * Hence proved.	A1	2.1
(b)	(i) (x =) 1, 8	B1	1.1b
	(ii) 8 is **not** a solution as log(4 – 8) cannot be found	B1	2.3
		(2)	
			(5 marks)

Notes:

(a)

B1: States or uses $2\log(4-x)=\log(4-x)^2$

M1: Correct attempt at eliminating the logs to form a quadratic equation in x.

Note that this may be implied by e.g. $\log\dfrac{(4-x)^2}{(x+8)}=0\Rightarrow(4-x)^2=x+8$

A1*: Proceeds to the given answer with at least one line where the $(4-x)^2$ has been multiplied out. There must be no errors or omissions but condone invisible brackets around the arguments of the logs e.g. allow $\log 16-8x+x^2$ for $\log(16-8x+x^2)$ and $\log x+8$ for $\log(x+8)$

Note we will allow a start of $(4-x)^2=x+8$ with no previous work for full marks.

Some examples of how to mark (a) in particular cases:

$$2\log(4-x)=\log(x+8)\Rightarrow\log(4-x)^2=\log(x+8)\Rightarrow\frac{\log(4-x)^2}{\log(x+8)}=1$$

$(4-x)^2$

$$2\log(4-x)=\log(x+8)\Rightarrow\log(4-x)^2-\log(x+8)=0\Rightarrow(4-x)^2-x-8=0$$

$$\Rightarrow 16-8x+x^2-x-8\Rightarrow x^2-9x+8=0$$

Scores B1M1A1

$$2\log(4-x)=\log(x+8)\Rightarrow\log(4-x)^2-\log(x+8)=0\Rightarrow\frac{\log(4-x)^2}{\log(x+8)}=0$$

$$\Rightarrow\frac{(4-x)^2}{(x+8)}=1\Rightarrow 16-8x+x^2=x+8\Rightarrow x^2-9x+8=0$$

Scores B1M0A0

(a) Alternative:

B1: Writes $x^2 - 9x + 8 = 0$ as $(4 - x)^2 - x - 8 = 0$ or equivalent

M1: Proceeds correctly to reach $\log(4-x)^2=\log(x+8)$

A1: Obtains $2\log(4-x)=\log(x+8)$ and makes a (minimal) conclusion e.g. hence proved, QED, #, square etc.

(b)

B1: Writes down (x =) 1, 8

B1: Chooses 8 (no follow through here) and gives a reason why it should be rejected by referring to logs and which log it is.

They must refer to the 8 as the required value but allow e.g. $x \neq 8$ and there must be a reference to $\log(4 - x)$ or log of lhs or log(–4) or the 4 – 8. Some acceptable reasons are: log(–4) can't be found/worked out/is undefined, log(–4) gives math error, log(–4) = n/a, lhs is log(negative) so reject, you can't do the log of a negative number which would happen with 4 – 8

Do **not** allow "you can't have a negative log" unless this is clarified further and do **not** allow "you get a math error" in isolation

There must be no contradictory statements.

Note that this is an independent mark but must have x = 8 (i.e. may have solved to get x = –1, 8 for first B mark)

Question	Scheme	Marks	AOs
4	${}^7C_4 a^3 (2x)^4$	M1	1.1b

	$\frac{7!}{4!3!}a^3 \times 2^4 = 15120 \Rightarrow a = ...$	dM1	2.1
	$a = 3$	A1	1.1b
		(3)	
(3 marks)			

Notes:

M1: For an attempt at the correct coefficient of x^4.
The coefficient must have

- the correct binomial coefficient
- the correct power of a
- 2 or 2^4 (may be implied)

May be seen within a full or partial expansion.

Accept ${}^7C_4 a^3(2x)^4$, $\frac{7!}{4!3!}a^3(2x)^4$, $\binom{7}{4}a^3(2x)^4$, $35a^3(2x)^4$, $560a^3x^4$, $\binom{7}{4}a^3 16x^4$ etc.

or ${}^7C_4 a^3 2^4$, $\frac{7!}{4!3!}a^3 2^4$, $\binom{7}{4}a^3 2^4$, $35a^3 2^4$, $560a^3$ etc.

or ${}^7C_3 a^3(2x)^4$, $\frac{7!}{4!3!}a^3(2x)^4$, $\binom{7}{3}a^3(2x)^4$, $35a^3(2x)^4$, $560a^3x^4$, $\binom{7}{3}a^3 16x^4$ etc.

or ${}^7C_3 a^3 2^4$, $\frac{7!}{4!3!}a^3 2^4$, $\binom{7}{3}a^3 2^4$, $35a^3 2^4$, $560a^3$

You can condone missing brackets around the "2x" so allow e.g. $\frac{7!}{4!3!}a^3 2x^4$

An alternative is to attempt to expand $a^7\left(1+\frac{2x}{a}\right)^7$ to give $a^7\left(...\frac{7\times6\times5\times4}{4!}\left(\frac{2x}{a}\right)^4...\right)$

Allow M1 for e.g. $a^7\left(...\frac{7\times6\times5\times4}{4!}\left(\frac{2x}{a}\right)^4...\right)$, $a^7\left(...\binom{7}{4}\left(\frac{2x}{a}\right)^4...\right)$, $a^7\left(...35\left(\frac{2x}{a}\right)^4...\right)$ etc.

but condone missing brackets around the $\frac{2x}{a}$

Note that ${}^7C_3$, $\binom{7}{3}$ etc. are equivalent to ${}^7C_4$, $\binom{7}{4}$ etc. and are equally acceptable.

If the candidate attempts $(a + 2x)(a + 2x)(a + 2x)$… etc. then it must be a complete method to reach the required term. Send to review if necessary.

dM1: For $"560"a^3 = 15120 \Rightarrow a = ...$ Condone slips on copying the 15120 but their "560" must be an attempt at ${}^7C_4\times 2$ or ${}^7C_4\times 2^4$ and must be attempting the <u>cube root</u> of $\frac{15120}{"560"}$. **Depends on the first mark**.

A1: a = 3 and no other values i.e. ±3 scores A0

Note that this is fairly common:

$${}^7C_4 a^3 2x^4 = 70a^3x^4 \Rightarrow 70a^3 = 15120 \Rightarrow a^3 = 216 \Rightarrow a = 6$$

and scores M1 dM1 A0

Question	Scheme	Marks	AOs
5	$15 - 2^{x+1} = 3\times 2^x$	B1	1.1b

	$\Rightarrow 15 - 2\times 2^x = 3\times 2^x \Rightarrow 2^x = 3$ or e.g. $\Rightarrow \frac{15}{2^x} - 2 = 3 \Rightarrow 2^x = 3$	M1	1.1b
	$2^x = 3 \Rightarrow x = \ldots$	dM1	1.1b
	$x = \log_2 3$	A1cso	1.1b
		(4)	
	Alternative		
	$y = 3\times 2^x \Rightarrow 2^x = \frac{y}{3} \Rightarrow y = 15 - 2\times\frac{y}{3}$	B1	1.1b
	$3y + 2y = 45 \Rightarrow y = 9 \Rightarrow 3\times 2^x = 9 \Rightarrow 2^x = 3$	M1	1.1b
	$2^x = 3 \Rightarrow x = \ldots$	dM1	1.1b
	$x = \log_2 3$	A1cso	1.1b
			(4 marks)

Notes:

B1: Combines the equations to reach $15 - 2^{x+1} = 3\times 2^x$ or equivalent e.g. $15 - 2^{x+1} - 3\times 2^x = 0$

M1: Uses $2^{x+1} = 2\times 2^x$ oe e.g. $\frac{2^{x+1}}{2^x} = 2$ to obtain an equation in 2^x and attempts to make 2^x the subject.

See scheme but e.g. $y = 2^x \Rightarrow 3\times 2^x = 15 - 2^{x+1} \Rightarrow 3y = 15 - 2y \Rightarrow y = \ldots$ is also possible

dM1: Uses logs correctly and proceeds to a value for x from an equation of the form $2^x = k$ where $k > 1$

e.g. $2^x = k \Rightarrow x = \log_2 k$

or $2^x = k \Rightarrow \log 2^x = \log k \Rightarrow x\log 2 = \log k \Rightarrow x = \ldots$

or $2^x = k \Rightarrow \ln 2^x = \ln k \Rightarrow x\ln 2 = \ln k \Rightarrow x = \ldots$

Depends on the first method mark

This may be implied if they go straight to decimals e.g. $2^x = 3$ so $x = 1.584..$ but you may need to check

A1cso: $x = \log_2 3$ or $\frac{\log 3}{\log 2}$ or $\frac{\ln 3}{\ln 2}$

Ignore any attempts to find the y-coordinate

Alternative

B1: Correct equation in y

M1: Solves their equation in y and attempts to make 2^x the subject.

dM1: Uses logs correctly and proceeds to a value for x from an equation of the form $2^x = k$ where $k > 1$

e.g. $2^x = k \Rightarrow x = \log_2 k$

or $2^x = k \Rightarrow \log 2^x = \log k \Rightarrow x\log 2 = \log k \Rightarrow x = \ldots$

or $2^x = k \Rightarrow \ln 2^x = \ln k \Rightarrow x\ln 2 = \ln k \Rightarrow x = \ldots$

Depends on the first method mark

This may be implied if they go straight to decimals e.g. $2^x = 3$ so $x = 1.584..$ but you may need to check

A1cso: $x = \log_2 3$ or $\frac{\log 3}{\log 2}$ or $\frac{\ln 3}{\ln 2}$

Ignore any attempts to find the y-coordinate

Question	Scheme	Marks	AOs

6(a)	$x^2+8x-3=(Ax+B)(x+2)+C$ or $Ax(x+2)+B(x+2)+C$ $\Rightarrow A=...,B=...,C=...$ **or** Long division of x^2+8x-3 by $x+2$: quotient $x+6$; x^2+2x; $6x-3$; $6x+12$; remainder -15	M1	1.1b
	Two of $A=1, B=6, C=-15$	A1	1.1b
	All three of $A=1, B=6, C=-15$	A1	1.1b
		(3)	
6(b)	$\int \frac{x^2+8x-3}{x+2}\,dx = \int x+6-\frac{15}{x+2}\,dx = ...-15\ln(x+2)$	M1	1.1b
	$=\frac{1}{2}x^2+6x-15\ln(x+2) \quad (+c)$	A1ft	1.1b
	$\int_0^6 \frac{x^2+8x-3}{x+2}\,dx = \left[\frac{1}{2}x^2+6x-15\ln(x+2)\right]_0^6$ = (18 + 36 – 15ln8) – (0 + 0 – 15ln2) = 18 + 36 – (15 – 45)ln2 or e.g. $18+36+15\ln\left(\frac{2}{8}\right)$	M1	2.1
	= 54 – 30 ln2	A1	1.1b
		(4)	
			(7 marks)

Notes:

(a)

M1: Multiplies by (x + 2) and attempts to find values for A, B and C e.g. by comparing coefficients or substituting values for x. If the method is unclear, at least 2 terms must be correct on rhs.

Or attempts to divide x^2+8x-3 by x + 2 and obtains a linear quotient and a constant remainder.

This mark may be implied by 2 correct values for A, B or C

A1: Two of A = 1, B = 6, C = –15. But note that **just** performing the division correctly is insufficient and they must clearly identify their A, B, C to score any accuracy marks.

A1: All three of A = 1, B = 6, C = –15

This is implied by stating $\frac{x^2+8x-3}{x+2}=x+6-\frac{15}{x+2}$ or within the integral in (b)

(b)

M1: Integrates an expression of the form $\frac{C}{x+2}$ to obtain $k\ln(x + 2)$.

Condone the omission of brackets around the "x + 2"

A1ft: Correct integration ft on their $Ax+B+\frac{C}{x+2}$, $(A,B,C\neq 0)$ The brackets should be present around the "x + 2" unless they are implied by subsequent work.

M1: Substitutes both limits 0 and 6 into an expression that contains an x or x^2 term or both and a ln term and subtracts either way round **WITH** fully correct log work to combine two log terms (but allow sign errors when removing brackets) leading to an answer of the form $a + b\ln c$ (a, b and c not necessarily integers)

e.g. if they expand to get –15ln8 – 15ln2 followed by -15ln16 and reach $a + b\ln c$ then allow the M mark

A1: 54 – 30 ln2 (Apply isw once a correct answer is seen)

Question	Scheme	Marks	AOs

7(a)	$\ln x \to \frac{1}{x}$	B1	1.1a
	Method to differentiate $\frac{4x^2+x}{2\sqrt{x}}$ – see notes	M1	1.1b
	E.g. $2\times\frac{3}{2}x^{\frac{1}{2}}+\frac{1}{2}\times\frac{1}{2}x^{-\frac{1}{2}}$	A1	1.1b
	$\frac{\mathrm{d}y}{\mathrm{d}x}=3\sqrt{x}+\frac{1}{4\sqrt{x}}-\frac{4}{x}=\frac{12x^2+x-16\sqrt{x}}{4x\sqrt{x}}$ *	A1*	2.1
		(4)	
(b)	$12x^2+x-16\sqrt{x}=0\Rightarrow 12x^{\frac{3}{2}}+x^{\frac{1}{2}}-16=0$	M1	1.1b
	E.g. $12x^{\frac{3}{2}}=16-\sqrt{x}$	dM1	1.1b
	$x^{\frac{3}{2}}=\frac{4}{3}-\frac{\sqrt{x}}{12}\Rightarrow x=\left(\frac{4}{3}-\frac{\sqrt{x}}{12}\right)^{\frac{2}{3}}$ *	A1*	2.1
		(3)	
(c)	$x_2=\sqrt[3]{\left(\frac{4}{3}-\frac{\sqrt{2}}{12}\right)^2}$	M1	1.1b
	x_2 = awrt 1.13894	A1	1.1b
	x = 1.15650	A1	2.2a
		(3)	
			(10 marks)

Notes:

(a)

B1: Differentiates $\ln x \to \frac{1}{x}$ seen or implied

M1: Correct method to differentiate $\frac{4x^2+x}{2\sqrt{x}}$:

Look for $\frac{4x^2+x}{2\sqrt{x}} \to ...x^{\frac{3}{2}}+...x^{\frac{1}{2}}$ being then differentiated to $Px^{\frac{1}{2}}+...$ **or** $...+Qx^{-\frac{1}{2}}$

Alternatively uses the quotient rule on $\frac{4x^2+x}{2\sqrt{x}}$.

Condone slips but if rule is not quoted expect $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)=\frac{2\sqrt{x}(Ax+B)-(4x^2+x)Cx^{-\frac{1}{2}}}{(2\sqrt{x})^2}(A,B,C>0)$

But a correct rule may be implied by their *u*, *v*, *u′*, *v′* followed by applying $\frac{vu'-uv'}{v^2}$ etc.

Alternatively uses the product rule on $(4x^2+x)(2\sqrt{x})^{-1}$

Condone slips but expect $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)=Ax^{-\frac{1}{2}}(Bx+C)+D(4x^2+x)x^{-\frac{3}{2}}(A,B,C>0)$

In general condone missing brackets for the M mark. If they quote $u = 4x^2 + x$ and $v = 2\sqrt{x}$ and don't make the differentiation easier, they can be awarded this mark for applying the correct rule. Also allow this mark if they quote the correct quotient rule but only have *v* rather than v^2 in the denominator.

A1: Correct differentiation of $\frac{4x^2+x}{2\sqrt{x}}$ although may not be simplified.

Examples: $\left(\frac{dy}{dx}\right)=\frac{2\sqrt{x}(8x+1)-(4x^2+x)x^{-\frac{1}{2}}}{(2\sqrt{x})^2}$, $\frac{1}{2}x^{-\frac{1}{2}}(8x+1)-\frac{1}{4}(4x^2+x)x^{-\frac{3}{2}}$, $2\times\frac{3}{2}x^{\frac{1}{2}}+\frac{1}{2}\times\frac{1}{2}x^{-\frac{1}{2}}$

A1*: Obtains $\frac{dy}{dx}=\frac{12x^2+x-16\sqrt{x}}{4x\sqrt{x}}$ via $3\sqrt{x}+\frac{1}{4\sqrt{x}}-\frac{4}{x}$ or a correct application of the quotient or product rule and with sufficient working shown to reach the printed answer.
<u>There must be no errors e.g. missing brackets.</u>

(b)

M1: Sets $12x^2+x-16\sqrt{x}=0$ and divides by $\sqrt{x}$ or equivalent e.g. divides by x and multiplies by $\sqrt{x}$

dM1: Makes the term in $x^{\frac{3}{2}}$ the subject of the formula
A1*: A correct and rigorous argument leading to the given solution.

Alternative - working backwards:

$$x=\left(\frac{4}{3}-\frac{\sqrt{x}}{12}\right)^{\frac{2}{3}}\Rightarrow x^{\frac{3}{2}}=\frac{4}{3}-\frac{\sqrt{x}}{12}\Rightarrow 12x^{\frac{3}{2}}=16-\sqrt{x}\Rightarrow 12x^2=16\sqrt{x}-x\Rightarrow 12x^2-16\sqrt{x}+x=0$$

M1: For raising to power of 3/2 both sides. dM1: Multiplies through by $\sqrt{x}$. A1: Achieves printed answer and makes a minimal comment e.g. tick, #, QED, true etc.

(c)

M1: Attempts to use the iterative formula with $x_1 = 2$. This is implied by sight of $x_2=\left(\frac{4}{3}-\frac{\sqrt{2}}{12}\right)^{\frac{2}{3}}$ or awrt 1.14

A1: x_2 = awrt 1.13894
A1: Deduces that x = 1.15650

<u>Via firstly integrating</u>

Question	Scheme	Marks	AOs

8	$f'(x)=6x^2+ax-23 \Rightarrow f(x)=2x^3+\frac{1}{2}ax^2-23x+c$	M1 A1	1.1b 1.1b
	$"c"=-12$	B1	2.2a
	$f(-4)=0 \Rightarrow 2\times(-4)^3+\frac{1}{2}a(-4)^2-23(-4)-12=0$	dM1	3.1a
	$a = \ldots$ (6)	dM1	1.1b
	$(f(x)=)2x^3+3x^2-23x-12$ Or Equivalent e.g. $(f(x)=)(x+4)(2x^2-5x-3)$ $(f(x)=)(x+4)(2x+1)(x-3)$	A1cso	2.1
		(6)	
			(6 marks)

Notes:

M1: Integrates $f'(x)$ with two correct indices. There is no requirement for the + c

A1: Fully correct integration (may be unsimplified). The + c must be seen (or implied by the –12)

B1: Deduces that the constant term is –12

dM1: Dependent upon having done some integration. It is for setting up a linear equation in a by using $f(-4)=0$

May also see long division attempted for this mark. Need to see a complete method leading to a remainder in terms of a which is then set = 0.

For reference, the quotient is $2x^2+\left(\frac{a}{2}-8\right)x+9-2a$ and the remainder is $8a-48$

May also use $(x + 4)(px^2 + qx + r) = 2x^3+\frac{1}{2}ax^2-23x-12$ and compare coefficients to find p, q and r and hence a. Allow this mark if they solve for p, q and r

Note that some candidates use 2f(x) which is acceptable and gives the same result if executed correctly.

dM1: Solves the linear equation in a or uses p, q and r to find a.

It is dependent upon having attempted some integration and used $f(\pm 4)=0$ or long division/comparing coefficients with $(x + 4)$ as a factor.

A1cso: For $(f(x)=)2x^3+3x^2-23x-12$ oe. Note that "f(x) =" does not need to be seen and ignore any "= 0"

Via firstly using factor

Question	Scheme	Marks	AOs
8 Alt	$f(x)=(x+4)(Ax^2+Bx+C)$	M1 A1	1.1b 1.1b
	$f(x)=Ax^3+(4A+B)x^2+(4B+C)x+4C \Rightarrow C=-3$	B1	2.2a
	$f'(x)=3Ax^2+2(4A+B)x+(4B+C)$ and $f'(x)=6x^2+ax-23$ $\Rightarrow A=\ldots$	dM1	3.1a
	Full method to get A, B and C	dM1	1.1b
	$f(x)=(x+4)(2x^2-5x-3)$	A1cso	2.1
		(6)	
			(6 marks)

Notes:

M1: Uses the fact that f(x) is a cubic expression with a factor of $(x + 4)$

A1: For $f(x) = (x + 4)(Ax^2 + Bx + C)$

B1: Deduces that $C = -3$

dM1: Attempts to differentiate either by product rule or via multiplication and compares to $f'(x) = 6x^2 + ax - 23$ to find A.

dM1: Full method to get A, B and C

A1cso: $f(x) = (x + 4)(2x^2 - 5x - 3)$ or $f(x) = (x + 4)(2x + 1)(x - 3)$

Question	Scheme	Marks	AOs
9(a)	$t = 0,\ \theta = 18 \Rightarrow 18 = A - B$ **or** $t = 10,\ \theta = 44 \Rightarrow 44 = A - Be^{-0.7}$	M1	3.1b

	$t=0,\ \theta=18 \Rightarrow 18=A-B$ **and** $t=10,\ \theta=44 \Rightarrow 44=A-B\mathrm{e}^{-0.7}$ **and** $\Rightarrow A=\ldots,\ B=\ldots$	M1	3.1a
	At least one of: A = 69.6, B = 51.6 but allow awrt 70/awrt 52	A1 **M1 on EPEN**	1.1b
	$\theta=69.6-51.6\mathrm{e}^{-0.07t}$	A1	3.3
		(4)	
(b)	The maximum temperature is "69.6"(°C) (according to the model) (The model has an) upper limit of "69.6"(°C) (The model suggests that) the boiling point is "69.6"(°C)	B1ft	3.4
	Model is not appropriate as 69.6(°C) is much lower than 78(°C)	B1ft	3.5a
		(2)	
			(6 marks)

Notes:

(a)

M1: Makes the first key step in the solution of the problem. Substitutes t = 0 and θ = 18 **or** t = 10 and θ = 44 into the equation of the model to obtain an equation connecting A and B.
Note that 18 = A – $B\mathrm{e}^0$ scores M0 unless 18 = A – B is seen or implied later.
If they do not obtain an equation in A and B using the first conditions e.g. they have 18 = A – 1 then they can score this mark if they substitute A = 19 directly into $44=A-B\mathrm{e}^{-0.7}$ as an equation in A and B is implied.

M1: Substitutes t = 0 and θ = 18 **and** t = 10 and θ = 44 to obtain 2 equations connecting A and B **and** then proceeds to solves their equations in A and B simultaneously to obtain values for both constants. Do not be too concerned with the processing as long as values for A and B are obtained.

A1(M1 on EPEN): For A = awrt 70 **or** B = awrt 52

A1: For $\theta=69.6-51.6\mathrm{e}^{-0.07t}$ **Must be a <u>fully correct equation as shown</u> but allow recovery if seen in (b).**
Note that some candidates evaluate e^0 as 0 and so obtain A = 18 and then write 44 = 18 – $B\mathrm{e}^{-0.7}$ and solve for B. Such attempts can score M1M0A0A0 only.

(b)

B1ft: Identifies A as the boiling point/maximum temperature in the model. Follow through their A.

B1ft: Makes a valid conclusion (valid/not valid, good/not good etc.) that refers to the 78 and includes a reference to a significant/large difference

Alternative provided their A < 78

B1ft: $\theta=69.6-51.6\mathrm{e}^{-0.07t}=78 \Rightarrow 51.6\mathrm{e}^{-0.07t}=69.6-78=-8.4$

$\Rightarrow \mathrm{e}^{-0.07t}=-\frac{7}{43}$ and $\ln\left(-\frac{7}{43}\right)$ and makes a reference to the fact that the equation cannot be solved or e.g. cannot take log of a negative number. You can condone numerical slips in the calculation.

B1ft: Model is not appropriate as 69.6(°C) is much lower than 78(°C)

Minimum for both marks: The model is not appropriate as "69.6"(°C) is much lower than 78(°C)

Note that these marks are not available if their equation is solvable. Note also that B0B1 is not possible.

Question	Scheme	Marks	AOs
10 (a)	cos 3A = cos (2A + A) = cos 2Acos A – sin 2Asin A	M1	3.1a
	$=\left(2\cos^2 A-1\right)\cos A-\left(2\sin A\cos A\right)\sin A$	dM1	1.1b
	$=\left(2\cos^2 A-1\right)\cos A-2\cos A\left(1-\cos^2 A\right)$	ddM1	2.1

	$= 4\cos^3 A - 3\cos A$*	A1*	1.1b
		(4)	
(b)	$1 - \cos 3x = \sin^2 x \Rightarrow \cos^2 x + 3\cos x - 4\cos^3 x = 0$	M1	1.1b
	$\Rightarrow \cos x\left(4\cos^2 x - \cos x - 3\right) = 0$ $\Rightarrow \cos x\left(4\cos x + 3\right)\left(\cos x - 1\right) = 0$ $\Rightarrow \cos x = ...$	dM1	3.1a
	Two of $-90°, 0, 90°$, awrt $139°$	A1	1.1b
	All four of $-90°, 0, 90°$, awrt $139°$	A1	2.1
		(4)	
			(8 marks)

Notes:

(a)

Allow a proof in terms of *x* rather than A

M1: Attempts to use the compound angle formula for cos(2*A* + *A*) or cos(*A* + 2*A*)
Condone a slip in sign

dM1: Uses correct double angle identities for cos 2*A* and sin 2*A*
$\cos 2A = 2\cos^2 A - 1$ must be used. If either of the other two versions are used expect to see an attempt to replace $\sin^2 A$ by $1 - \cos^2 A$ at a later stage.
Depends on previous mark.

ddM1: Attempts to get all terms in terms of cos *A* using correct and appropriate identities.
Depends on both previous marks.

A1*: A completely correct and rigorous proof including correct notation, no mixed variables, missing brackets etc.

Alternative right to left is possible:

$4\cos^3 A - 3\cos A = \cos A\left(4\cos^2 A - 3\right) = \cos A\left(2\cos^2 A - 1 + 2\left(1 - \sin^2 A\right) - 2\right) = \cos A\left(\cos 2A - 2\sin^2 A\right)$

$= \cos A\cos 2A - 2\sin A\cos A\sin A = \cos A\cos 2A - \sin 2A\sin A = \cos(2A + A) = \cos 3A$

Score M1: For $4\cos^3 A - 3\cos A = \cos A\left(4\cos^2 A - 3\right)$

dM1: For $\cos A\left(2\cos^2 A - 1 + 2\left(1 - \sin^2 A\right) - 2\right)$ (Replaces $4\cos^2 A - 1$ by $2\cos^2 A - 1$ and $2\left(1 - \sin^2 A\right)$)

ddM1: Reaches $\cos A\cos 2A - \sin 2A\sin A$

A1: $\cos(2A + A) = \cos 3A$

(b)

M1: For an attempt to produce an equation just in cos *x* using both part (a) and the identity $\sin^2 x = 1 - \cos^2 x$
Allow one slip in sign or coefficient when copying the result from part (a)

dM1: Dependent upon the preceding mark. It is for taking the cubic equation in cos *x* and making a valid attempt to solve. This could include factorisation or division of a cos *x* term followed by an attempt to solve the 3 term quadratic equation in cos *x* to reach at least one non zero value for cos *x*.
May also be scored for solving the cubic equation in cos *x* to reach at least one non zero value for cos *x*.

A1: Two of $-90°, 0, 90°$, awrt $139°$ **Depends on the <u>first</u> method mark**.

A1: All four of $-90°, 0, 90°$, awrt $139°$ with no extra solutions offered within the range.

Note that this is an alternative approach for obtaining the cubic equation in (b):

$$1-\cos 3x=\sin^2 x \Rightarrow 1-\cos 3x=\frac{1}{2}(1-\cos 2x)$$

$$\Rightarrow 2-2\cos 3x=1-\cos 2x$$

$$\Rightarrow 1=2\cos 3x-\cos 2x$$

$$\Rightarrow 1=2\left(4\cos^3 x-3\cos x\right)-\left(2\cos^2 x-1\right)$$

$$\Rightarrow 0=4\cos^3 x-3\cos x-\cos^2 x$$

The M1 will be scored on the penultimate line when they use part (a) and use the correct identity for cos 2*x*

Question	Scheme	Marks	AOs
11(a)	$x=-4$ **or** $y=-5$	B1	1.1b

	$P(-4,-5)$	B1	2.2a
		(2)	
(b)	$3x+40=-2(x+4)-5 \Rightarrow x=...$	M1	1.1b
	$x=-10.6$	A1	2.1
		(2)	
(c)	$a>2$	B1	2.2a
	$y=ax \Rightarrow -5=-4a \Rightarrow a=\frac{5}{4}$	M1	3.1a
	$\{a:a\leqslant 1.25\}\cup\{a:a>2\}$	A1	2.5
		(3)	
			(7 marks)

Notes:

(a)

B1: One correct coordinate. Either $x = –4$ or $y = –5$ or (–4, …) or (…,–5) seen.

B1: Deduces that $P(-4,-5)$ Accept written separately e.g. $x = –4$, $y = –5$

(b)

M1: Attempts to solve $3x+40=-2(x+4)-5 \Rightarrow x=...$ Must reach a value for *x*.

You may see the attempt crossed out but you can still take this as an attempt to solve the required equation.

A1: $x = –10.6$ oe e.g. $-\frac{53}{5}$ only. If other values are given, e.g. $x = –37$ they must be rejected or the $-\frac{53}{5}$ clearly chosen as their answer. Ignore any attempts to find *y*.

Alternative by squaring:

$$3x+40=2|x+4|-5 \Rightarrow 3x+45=2|x+4| \Rightarrow 9x^2+270x+2025=4\left(x^2+8x+16\right)$$

$$\Rightarrow 5x^2+238x+1961=0 \Rightarrow x=-37,-\frac{53}{5}$$

M1 for isolating the $|x+4|$, squaring both sides and solving the resulting quadratic

A1 for selecting the $-\frac{53}{5}$

Correct answer with no working scores both marks.

(c)

B1: Deduces that $a > 2$

M1: Attempts to find a value for *a* using their $P(-4,-5)$

Alternatively attempts to solve $ax = 2(x + 4) – 5$ and $ax = 2(x + 4) – 5$ to obtain a value for *a*.

A1: Correct range in acceptable set notation.

Examples:

$\{a:a\leqslant 1.25\}\cup\{a:a>2\}$

$\{a:a\leqslant 1.25\},\ \{a:a>2\}$

$\{a:a\leqslant 1.25 \text{ or } a>2\}$

$\{a:a\leqslant 1.25,\ a>2\}$

$(-\infty,1.25]\cup(2,\infty)$

$(-\infty,1.25],\ (2,\infty)$

Question	Scheme	Marks	AOs
12(a)(i)	$y\times\frac{\mathrm{d}x}{\mathrm{d}t}=5\sin 2t\times 6\cos t$ or $5\times 2\sin t\cos t\times 6\cos t$	M1	1.2

	(Area =) $\int 5\sin 2t \times 6\cos t\,dt = \int 5\times 2\sin t\cos t \times 6\cos t\,dt$ or $\int 5\sin 2t \times 6\cos t\,dt = \int 60\sin t\cos^2 t\,dt$	dM1	1.1b
	(Area =) $\int_0^{\frac{\pi}{2}} 60\sin t\cos^2 t\,dt$ *	A1*	2.1*
		(3)	
(a)(ii)	$\int 60\sin t\cos^2 t\,dt = -20\cos^3 t$	M1 A1	1.1b 1.1b
	Area = $\left[-20\cos^3 t\right]_0^{\frac{\pi}{2}} = 0-(-20) = 20$ *	A1*	2.1
		(3)	
(b)	$5\sin 2t = 4.2 \Rightarrow \sin 2t = \frac{4.2}{5}$	M1	3.4
	$t = 0.4986...,1.072...$	A1	1.1b
	Attempts to finds the x values at both t values	dM1	3.4
	$t = 0.4986... \Rightarrow x = 2.869...$ $t = 1.072 \Rightarrow x = 5.269...$	A1	1.1b
	Width of path = 2.40 metres	A1	3.2a
		(5)	
			(11 marks)

Notes:

(a)(i)

M1: Attempts to multiply y by $\frac{dx}{dt}$ to obtain $A\sin 2t\cos t$ but may apply $\sin 2t = 2\sin t\cos t$ here

dM1: Attempts to use $\sin 2t = 2\sin t\cos t$ within an integral which may be implied by

e.g. $A\int \sin 2t \times \cos t\,dt = \int k\sin t\cos^2 t\,dt$

A1*: Fully correct work leading to the given answer.

This must include $\sin 2t = 2\sin t\cos t$ or e.g. $5\sin 2t = 10\sin t\cos t$ seen <u>explicitly</u> in their proof and a correct intermediate line that includes an integral sign and the "dt"

Allow the limits to just "appear" in the final answer e.g. working need not be shown for the limits.

(a)(ii)

M1: Obtains $\int 60\sin t\cos^2 t\,dt = k\cos^3 t$. This may be attempted via a substitution of $u = \cos t$ to obtain $\int 60\sin t\cos^2 t\,dt = ku^3$

A1: Correct integration $-20\cos^3 t$ or equivalent e.g. $-20u^3$

A1*: Rigorous proof with all aspects correct including the correct limits and the $0-(-20)$ and

not <u>just:</u> $-20\cos^3\frac{\pi}{2} - \left(-20\cos^3 0\right) = 20$

(b)

M1: Uses the given model and attempts to find value(s) of t when $\sin 2t = \frac{4.2}{5}$. Look for $2t = \sin^{-1}\frac{4.2}{5} \Rightarrow t = ...$

A1: At least one correct value for t, correct to 2 dp. FYI $t = 0.4986...,1.072...$ or in degrees $t = 28.57..., 61.42...$

dM1: Attempts to find **TWO** distinct values of x when sin 2t = $\frac{4.2}{5}$. Condone poor trig work and allow this mark if 2 values of x are attempted from 2 values of t.

A1: Both values correct to 2 dp. NB x = 2.869…, 5.269…

Or may take Cartesian approach

$$5\sin 2t = 4.2 \Rightarrow 10\sin t\cos t = 4.2 \Rightarrow 10\frac{x}{6}\sqrt{1-\frac{x^2}{36}} = 4.2 \Rightarrow x^4 - 36x^2 + 228.6144 = 0 \Rightarrow x = 2.869...,5.269...$$

M1: For converting to Cartesian form A1: Correct quartic M1: Solves quartic A1: Correct values

A1: 2.40 metres or 240 cm

Allow awrt 2.40 m or allow 2.4m (not awrt 2.4 m) and allow awrt 240 cm. **Units are required**.

Question	Scheme	Marks	AOs
13(a)	$k = e^2$ **or** $x \neq e^2$	B1	2.2a
		(1)	

(b)	$g'(x)=\dfrac{(\ln x-2)\times\frac{3}{x}-(3\ln x-7)\times\frac{1}{x}}{(\ln x-2)^2}=\dfrac{1}{x(\ln x-2)^2}$ or $g'(x)=\dfrac{d}{dx}\left(3-(\ln(x)-2)^{-1}\right)=(\ln x-2)^{-2}\times\dfrac{1}{x}=\dfrac{1}{x(\ln x-2)^2}$ or $g'(x)=(\ln x-2)^{-1}\times\dfrac{3}{x}-(3\ln x-7)(\ln x-2)^{-2}\times\dfrac{1}{x}=\dfrac{1}{x(\ln x-2)^2}$	M1 A1	1.1b 2.1
	As $x > 0$ (or $1/x > 0$) **AND** $\ln x - 2$ is squared so $g'(x) > 0$	A1cso	2.4
		(3)	
(c)	Attempts to solve either $3\ln x - 7 \ldots 0$ or $\ln x - 2 \ldots 0$ or $3\ln a - 7 \ldots 0$ or $\ln a - 2 \ldots 0$ where … is "=" or ">" to reach a value for x or a but may be seen as an inequality e.g. $x > \ldots$ or $a > \ldots$	M1	3.1a
	$0 < a < e^2, \quad a > e^{\frac{7}{3}}$	A1	2.2a
		(2)	
			(6 marks)

Notes:

(a)

B1: Deduces $k = e^2$ **or** $x \neq e^2$ Condone k = awrt 7.39 or $x \neq$ awrt 7.39

(b)

M1: Attempts to differentiate via the quotient rule and with $\ln x \to \dfrac{1}{x}$ so allow for:

$$\frac{d}{dx}(g(x))=\frac{(\ln x-2)\times\frac{\alpha}{x}-(3\ln x-7)\times\frac{\beta}{x}}{(\ln x-2)^2},\ \beta>0$$

But a correct rule may be implied by their *u*, *v*, *u′*, *v′* followed by applying $\dfrac{vu'-uv'}{v^2}$ etc.

Alternatively attempts to write $g(x)=\dfrac{3\ln(x)-7}{\ln(x)-2}=3-(\ln(x)-2)^{-1}$ and attempts the chain rule so allow for:

$$3-(\ln(x)-2)^{-1}\to(\ln(x)-2)^{-2}\times\frac{\alpha}{x}$$

Alternatively writes $g(x)=(3\ln(x)-7)(\ln(x)-2)^{-1}$ and attempts the product rule so allow for:

$$g'(x)=(\ln x-2)^{-1}\times\frac{\alpha}{x}-(3\ln x-7)(\ln x-2)^{-2}\times\frac{\beta}{x}$$

In general condone missing brackets for the M mark. E.g. if they quote $u = 3\ln x - 7$ and $v = \ln x - 2$ and don't make the differentiation easier, they can be awarded this mark for applying the correct rule. Also allow this mark if they quote the correct quotient rule but only have v rather than v^2 in the denominator.

A1: $\dfrac{1}{x(\ln x-2)^2}$ **Allow** $\dfrac{\frac{1}{x}}{(\ln x-2)^2}$ **i.e. we need to see the numerator simplified to $1/x$**

Note that some candidates establish the correct numerator and correct denominator independently and provided they obtain the correct expressions, this mark can be awarded.
But allow a correctly expanded denominator.

A1cso: States that as $x > 0$ **AND** $\ln x - 2$ is squared so $g'(x) > 0$

(c)

M1: Attempts to solve either 3ln x – 7 = 0 or ln x – 2 = 0 or using inequalities e.g. 3ln x – 7 > 0

A1: $0 < a < e^2, \ a > e^{\frac{7}{3}}$

Question	Scheme	Marks	AOs
14 (a)	C is $(x-r)^2+(y-r)^2=r^2$ or $x^2+y^2-2rx-2ry+r^2=0$	B1	2.2a
	$y=12-2x,\ x^2+y^2-2rx-2ry+r^2=0$ $\Rightarrow x^2+(12-2x)^2-2rx-2r(12-2x)+r^2=0$ or	M1	1.1b

	$y=12-2x,\ \ (x-r)^2+(y-r)^2=r^2$ $\Rightarrow (x-r)^2+(12-2x-r)^2=r^2$		
	$x^2+144-48x+4x^2-2rx-24r+4rx+r^2=0$ $\Rightarrow 5x^2+(2r-48)x+\left(r^2-24r+144\right)=0\ \ *$	A1*	2.1
		(3)	
(b)	$b^2-4ac=0\Rightarrow (2r-48)^2-4\times5\times\left(r^2-24r+144\right)=0$	M1	3.1a
	$r^2-18r+36=0$ or any multiple of this equation	A1	1.1b
	$\Rightarrow (r-9)^2-81+36=0\Rightarrow r=...$	dM1	1.1b
	$r=9\pm3\sqrt{5}$	A1	1.1b
		(4)	
			(7 marks)

Notes:

(a)
B1: Deduces the correct equation of the circle
M1: Attempts to form an equation with terms of the form x^2, x, r^2, and xr **only** using $y=12\pm2x$ and their circle equation which must be of an appropriate form. I.e. includes or implies an x^2, y^2, r^2 such as $x^2+y^2=r^2$
If their circle equation starts off as e.g. $(x\pm a)^2+(y\pm b)^2=r^2$ then the B mark and the M mark can be awarded when the "a" and "b" are replaced by r or $-r$ as appropriate for their circle equation.
A1*: Uses correct and accurate algebra leading to the given solution.

(b)
M1: Attempts to use $b^2-4ac...0$ o.e. with $a=5, b=2r-48, c=r^2-24r+144$ and where ... is "=" or any inequality
Allow minor slips when copying the a, b and c provided it does not make the work easier and allow **their** a, b and c if they are similar expressions.
FYI $(2r-48)^2-4\times5\times\left(r^2-24r+144\right)=4r^2-192r+2304-20r^2+480r-2880=-16r^2+288r-576$
A1: Correct quadratic **equation** in r (or inequality). Terms need not be all one side but must be collected.
E.g. allow $r^2-18r=-36$ and allow any multiple of this equation (or inequality).
dM1: Correct attempt to solve their 3TQ in r. Dependent upon previous M
A1: Careful and accurate work leading to both answers in the required form (must be simplified surds)

Question	Scheme	Marks	AOs
15(a)	$S_n=a+ar+ar^2+........+ar^{n-1}$	B1	1.2
	$rS_n=ar+ar^2+ar^3+........+ar^n\Rightarrow S_n-rS_n=...$	M1	2.1
	$S_n-rS_n=a-ar^n$	A1	1.1b

	$S_n(1-r)=a(1-r^n) \Rightarrow S_n=\dfrac{a(1-r^n)}{(1-r)}$*	A1*	2.1
		(4)	
(b)	$\dfrac{a(1-r^{10})}{1-r}=4\times\dfrac{a(1-r^5)}{1-r}$ or $4\times\dfrac{a(1-r^{10})}{1-r}=\dfrac{a(1-r^5)}{1-r}$ Equation in r^{10} and r^5 (and possibly 1 – *r*)	M1	3.1a
	$1-r^{10}=4(1-r^5)$	A1	1.1b
	$r^{10}-4r^5+3=0 \Rightarrow (r^5-1)(r^5-3)=0 \Rightarrow r^5=...$ or e.g. $1-r^{10}=4(1-r^5) \Rightarrow (1-r^5)(1+r^5)=4(1-r^5) \Rightarrow r^5=...$	dM1	2.1
	$r=\sqrt[5]{3}$ oe only	A1	1.1b
		(4)	
			(8 marks)

Notes:

(a)

B1: Writes out the sum or lists terms. Condone the omission of *S*.
The sum must include the first and last terms and (at least) two other correct terms and no incorrect terms e.g. ar^n
Note that the sum may be seen embedded within their working.

M1: For the key step in attempting to multiply the first series by *r* and subtracting.

A1: $S_n - rS_n = a - ar^n$ either way around but condone one side being prematurely factorised (but not both) **following correct work but this could follow B0 if insufficient terms were shown.**

A1*: Depends on all previous marks. Proceeds to given result showing all steps including seeing both sides unfactorised at some point in their working.

Note: If terms are <u>listed</u> rather than <u>added</u> then allow the first 3 marks if otherwise correct but withhold the final mark.

(b)

M1: For the correct strategy of producing an equation in just r^{10} and r^5 (and possibly (1 – *r*)) with the "4" on either side using the result from part (a) and makes progress to at least cancel through by *a*
Some candidates retain the "1 – *r*" and start multiplying out e.g. $(1-r)(1-r^{10})$ and this mark is still available as long as there is progress in cancelling the "*a*".

A1: Correct equation with the *a* and the 1 – *r* cancelled. Allow any correct equation in just r^5 and r^{10}

dM1: Depends on the first M. Solves as far as $r^5=...$ by solving a 3 term quadratic in r^5 by a valid method – see general guidance or by difference of 2 squares – see above

A1: $r=\sqrt[5]{3}$ oe only. The solution *r* = 1 if found must be rejected here.

> **(b) Note**: For candidates who use $S_5 = 4S_{10}$ expect to see:
>
> $4\times\dfrac{a(1-r^{10})}{1-r}=\dfrac{a(1-r^5)}{1-r} \Rightarrow 4(1-r^{10})=(1-r^5)$ M1A0
>
> $4r^{10}-r^5-3=0 \Rightarrow (4r^5+3)(r^5-1)=0 \Rightarrow r^5=...$ or $4(1-r^5)(1+r^5)=(1-r^5) \Rightarrow r^5=...$dM1A0

<u>Example for (a):</u>

a.

$S_n = a + ar + ar^2 + ar^3 ... ar^{n-2} + ar^{n-1}$

$rS_n = ar + ar^2 + ar^3 + ar^4 ... ar^{n-1} + ar^n$

$S_n - rS_n = a(1-r^n)$

$S_n(1-r) = a(1-r^n)$

$S_n = a(1-r^n)$

This scores B1M1A1A0:

B1: Writes down the sum including first and last terms and at least 2 other correct terms and no incorrect terms

M1: Multiplies by *r* and subtracts

A1: Correct equation (we allow one side to be prematurely factorised)

A0: One side was prematurely factorised

Question	Scheme	Marks	AOs
16	NB any natural number can be expressed in the form: $3k$, $3k+1$, $3k+2$ or equivalent e.g. $3k-1$, $3k$, $3k+1$		
	Attempts to square **any two** distinct cases of the above	M1	3.1a
	Achieves accurate results and makes a valid comment for **any two** of the possible three cases: E.g. $(3k)^2 = 9k^2 \left(= 3\times 3k^2\right)$ is a multiple of 3	A1 **M1 on EPEN**	1.1b

	$(3k+1)^2 = 9k^2 + 6k + 1 = 3\times(3k^2 + 2k) + 1$ is one more than a multiple of 3 $(3k+2)^2 = 9k^2 + 12k + 4 = 3\times(3k^2 + 4k + 1) + 1$ $\left(\text{or } (3k-1)^2 = 9k^2 - 6k + 1 = 3\times(3k^2 - 2k) + 1\right)$ is one more than a multiple of 3		
	Attempts to square in all 3 distinct cases. E.g. attempts to square $3k$, $3k + 1$, $3k + 2$ or e.g. $3k - 1$, $3k$, $3k + 1$	M1 **A1 on EPEN**	2.1
	Achieves accurate results for all three cases and gives a minimal conclusion (allow tick, QED etc.)	A1	2.4
		(4)	
			(4 marks)

Notes:

M1: Makes the key step of attempting to write the natural numbers in any 2 of the 3 distinct forms or equivalent expressions, as shown in the mark scheme, and attempts to square these expressions.

A1(M1 on EPEN): Successfully shows for 2 cases that the squares are either a multiple of 3 or 1 more than a multiple of 3 using algebra. This must be made explicit e.g. reaches $3\times(3k^2 + 2k) + 1$ and makes a statement that this is one more than a multiple of 3 but also allow other rigorous arguments that reason why $9k^2 + 6k + 1$ is one more than a multiple of 3 e.g. "$9k^2$ is a multiple of 3 and $6k$ is a multiple of 3 so $9k^2 + 6k + 1$ is one more than a multiple of 3"

M1(A1 on EPEN): Recognises that all natural numbers can be written in one of the 3 distinct forms or equivalent expressions, as shown in the mark scheme, and attempts to square in all 3 cases.

A1: Successfully shows for all 3 cases that the squares are either a multiple of 3 or 1 more than a multiple of 3 using algebra and makes a conclusion